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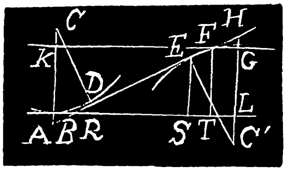
Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $\tan OCD=\tan EC'G=m$, $OC=CD=C'G=C'E=R$, $DE=L$, $KO=LG=p$. The equation to the line AH is $y=mx-b$, where O is origin and $b=OA$.
$CA=CD\sec OCD$. $\therefore R+b=R\sqrt{(1+m^2)}$.
$\therefore y=mx+R-R\sqrt{(1+m^2)}$.
When $y=0$, $x=R[\sqrt{(1+m^2)}-1]/m$,
When $y=p$, $x=[p+R\sqrt{(1+m^2)}-R]/m=x_1$.
$\therefore OB=BD=EF=FG=R[\sqrt{(1+m^2)}-1]/m$.
Since $BF=\sqrt{[BT^2+TF^2)}=\sqrt{[(x_1-x)^2+p^2]}$,
$\therefore L+2R[\sqrt{(1+m^2)}-1]/m=\sqrt{[(p/m)^2+p^2]}=(p/m)\sqrt{(1+m^2)}$.
$\therefore [(2R-p)^2-L^2]m^2+4RLm=4Rp-p^2$. This determines m.
$OL=OT+TL=x+x_1=[p+2R\sqrt{(1+m)^2}-2R]/m$.
$BR=BD\cos OCD=x/\sqrt{(1+m^2)}=R[\sqrt{(1+m^2)}-1]/[m\sqrt{(1+m^2)}]$.
$RS=BT-2BR=p/m-2R[\sqrt{(1+m^2)}-1]/[m\sqrt{(1+m^2)}]$.
$p=62$, $L=100$, $R=604$.
$\therefore 325829m^2+60400m=36487$ or $m=.254559$.
$\therefore \angle OCD=14^\circ\ 16'\ 52.6''=$central angles.
$OL=394.9$ feet, $RS=96.9$ feet.



CALCULUS.

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112. **Proposed by J. SCHEFFER, A. M., Hagerstown, Md.**

A sphere of radius r is pierced by a cylinder radius $\frac{1}{2}r$ so that the cylinder just grazes the center of the sphere. Find volume removed; the lateral surface and the spherical surface removed.

Solution by L. C. WALKER, A. M., Assistant Professor of Mathematics, Leland Stanford Jr. University, Palo Alto, Cal.; G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.; and the PROPOSER.

Taking the center of the sphere for coördinates, we have for the equations of sphere and cylinder, respectively,

$$x^2+y^2+z^2=r^2,\quad y^2=rx-x^2.$$

Therefore volume removed, $V=4\int_0^r dx\int_0^{\sqrt{(rx-x^2)}} zdy$

$$=2\int_0^r\left[(r^2-x^2)\sin^{-1}\sqrt{\frac{x}{r+x}}dx+\sqrt{(rx)(r-x)}dx\right]=\frac{3\pi-4}{9}.2r^3.$$

The lateral surface $S=2\int z\partial s$, s being an arc of the base circle of the cylinder. From $y^2=rx-x^2$,

$$\frac{dy}{dx}=\frac{r-2x}{2\sqrt{(rx-x^2)}},\quad \frac{dz^2}{dx^2}=1+\left(\frac{dy}{dx}\right)^2=\frac{r^2}{4(rx-x^2)},\quad z^2=r^2-x^2-y^2=r^2-rx.$$

$$\therefore\ L=2r\sqrt{r}\int_0^r\frac{dx}{\sqrt{x}}=4r^2.$$

The spherical surface removed, $S=4\int_0^r dx\int_0^{\sqrt{(rx-x^2)}}dy\sqrt{1+\left(\frac{dz}{dx}\right)^2+\left(\frac{dz}{dy}\right)^2}$

$$=4r\int_0^r dx\int_0^{\sqrt{(rx-x^2)}}\frac{dy}{\sqrt{(r^2-x^2-y^2)}}=4r\int_0^r\sin^{-1}\sqrt{\frac{x}{r+x}}.dx=(\pi-2)2r^2.$$

Note. The results of Problem 6 in Byerly's *Integral Calculus*, page 282, 2nd Ed. (1898) are correct, if b represents the diameter of the cylindrical hole instead of the radius.

Also solved by *H. C. WHITAKER.*

113. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

At what rate per unit of time are the roots of the equation ($x^2+px+q=0$) changing, if $p=mq$ and q varies uniformly at the rate of 1/12 per unit of time, when $p=12$ and m remains constant?

Solution by H. C. WHITAKER, A. M., Ph. D., Manual Training School, Philadelphia, Pa.; J. SCHEFFER, A. M., Hagerstown, Md.; and G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.

The roots of the given equation are

$$x=\tfrac{1}{2}[-mq\pm\sqrt{(m^2q^2-4q)}],$$

$$dx=\tfrac{1}{2}[-m\pm(m^2q-2)(m^2q^2-4q)^{-\frac{1}{2}}]dq.$$

When $q=12\div m$ and $dq=\frac{1}{12}$,

$$dx=\tfrac{1}{24}\left[-m\pm\frac{(6m-1)\sqrt{m}}{2\sqrt{(9m-3)}}\right].$$

Also solved by *WILLIAM HOOVER.*

114. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

If two concentric ellipses have equal axes inclined at an angle ω, their common area is

$$A=2ab\tan^{-1}\left(\frac{2ab}{(a^2-b^2)\sin\omega}\right).$$

Solution by J. SCHEFFER, A. M., Hagerstown, Md., and J. B. GREGG, Senecaville, Ohio.

Let AB be the semi-major axis of one ellipse, and AB' that of the other, MAC and NAD being common diameters. F is the intersection of AB with the second ellipse, and E and G are the intersections of major axis of the second ellipse with the first.

Let angle $B'AB=\omega$. It can be easily shown that angle $CAF=\frac{1}{2}\omega$, and angle DAF=angle $DAG=\frac{1}{2}\pi-\frac{1}{2}\omega$.